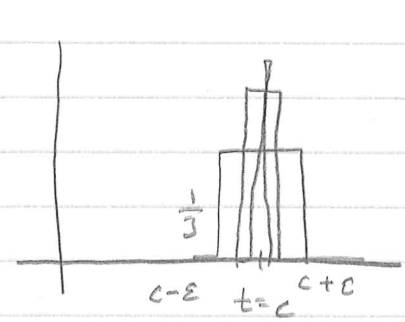
DELTA FUNCTION

$\lim_{\varepsilon \to 0} \mathcal{L}\{h_\varepsilon(t)\} = e^{-cs}$

in particular if $c=0$, then

$\lim_{\varepsilon \to 0} \mathcal{L}\{h_\varepsilon(t)\} = e^{0s} = 1$



We defined

$\delta_c(t)$ SUCH THAT

$$h_\varepsilon(t) = \begin{cases} \frac{1}{\varepsilon}, & c-\varepsilon \le t \le c+\varepsilon; \\ 0, & \text{otherwise} \end{cases}$$

$$\boxed{\mathcal{L}\{\delta_c(t)\} = e^{-cs}}$$

and we think of $\delta_c(t)$ as $\lim_{\varepsilon \to 0} h_\varepsilon(t)$.

an "impulse at $t=c$"

$$\delta_c(t) = \begin{cases} \infty, & t=c \\ 0, & t \neq c \end{cases}$$

Ex $y'' + 4y' + 40y = \delta_0(t)$, $y(0)=0$, $y'(0)=0$

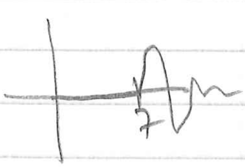
$(s^2+4s+40)\mathcal{L}\{y\} = \mathcal{L}\{\delta_0(t)\} = 1$

$\mathcal{L}\{y\} = \frac{1}{s^2+4s+40} = G(s)$ TRANSFER FUNCTION!

$\Rightarrow y = \mathcal{L}^{-1}\left\{\frac{1}{s^2+4s+40}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+36}\right\}$

$y = \frac{1}{6}e^{-2t}\sin(6t) = g(t) =$ "impulse response"!

$y'' + 4y' + 40y = \delta_7(t)$ $\quad y(0)=0 \quad y'(0)=0$



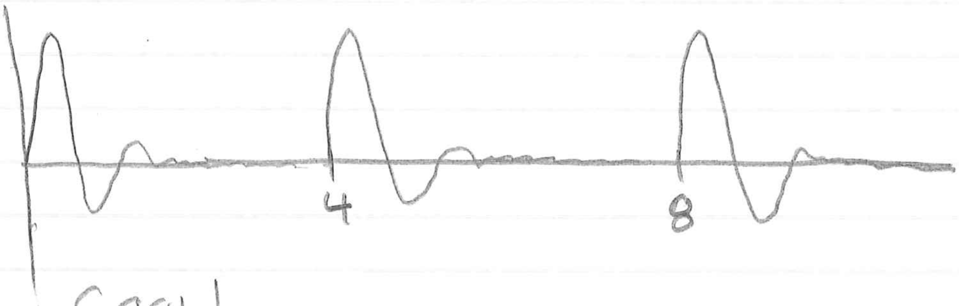
$\mathcal{L}\{y\} = e^{-7s}\frac{1}{s^2+4s+40}$

$y = \mathcal{L}^{-1}\left\{e^{-7s}\frac{1}{(s+2)^2+36}\right\} = u_7(t)\,\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+36}\right\}\Big|_{t \to t-7}$

$y = u_7(t)\frac{1}{6}e^{-2(t-7)}\sin(6(t-7)) = \begin{cases} 0, & t<7; \\ & t \ge 7. \end{cases}$

$$y'' + 4y' + 40y = \delta_0(t) + \delta_4(t) + \delta_8(t) \qquad y(0)=0,\ y'(0)=0$$

$$y = g(t) + u_4(t)\,g(t-4) + u_8(t)\,g(t-8)$$



COOL!

$\boxed{\text{HW 10/4}}$

$$y'' + 8y' + 25y = \delta_4(t)\ ,\ \overbrace{y(0)=4\ ,\ y'(0)=-22}^{\text{what now?}}$$

$$(s^2\mathcal{L}\{y\} - 4s - -22) + 8(s\mathcal{L}\{y\} - 4) + 25\mathcal{L}\{y\} = e^{-4s}$$

$$(s^2 + 8s + 25)\mathcal{L}\{y\} - 4s + 22 - 32 = e^{-4s}$$

$$\mathcal{L}\{y\} = \frac{4s+10}{s^2+8s+25} + e^{-4s}\frac{1}{s^2+8s+25}$$

$$s^2 + 8s + 16 - 16 + 25 = (s+4)^2 + 9$$

$$\frac{4s+10}{(s+4)^2+9} = \frac{A(s+4)+B}{(s+4)^2+9} \Rightarrow 4s+10 = As + 4A + B$$

$$A = 4 \ \&\ 4A + B = 10$$

$$B = -6$$

$$\boxed{y = \mathcal{L}^{-1}\left\{\frac{4(s+4)}{(s+4)^2+9} - \frac{6}{(s+4)^2+9}\right\} + u_4(t)\,\mathcal{L}^{-1}\left\{\frac{1}{(s+4)^2+9}\right\}\bigg|_{t\to t-4}}$$

$$\boxed{= 4e^{-4t}\cos(3t) - \frac{6}{3}e^{-4t}\sin(3t) + u_4(t)\frac{1}{3}e^{-4(t-4)}\sin(3(t-4))}$$

Recall the fact about the "impulse response"
namely
if $g(t) = \mathcal{L}^{-1}\left\{\frac{1}{ms^2+\gamma s+k}\right\}$

then the sol'n to
$$my'' + \gamma y' + ky = f(t), \quad y(0)=0, \; y'(0)=0$$

is

impulse response

forcing function

$$y = \int_0^t g(t-s)\, f(s)\, ds. \quad = g * f$$

convolution

HAND OUT LAST TWO PAGES OF MOST RECENT FINAL & DISCUSS

FINAL:

2-3 PAGES FROM EXAM 1 MATERIAL (1st order)
- Integrating Factor Method
- Separable equations
- Equilibrium sol'ns
- Applications from HW & CLASS
- Approximating

2-3 PAGES FROM EXAM 2 MATERIAL
- Homogeneous Sol'ns to $ay''+by'+cy=0$
- Particular Sol'ns to $ay''+by'+cy=f(t)$ "Undetermined coefficients"
- MASS-SPRING ANALYSIS & TERMS
- INTERPRETING SOL'NS / GRAPHS

2-3 PAGES FROM LAPLACE TRANSFORMS
- $\mathcal{L}\{f(t)\} = ?$
- $\mathcal{L}^{-1}\{F(s)\} = ?$
- shifting & step functions
- $\mathcal{L}\{y'\} = s\mathcal{L}\{y\} - y(0)$ $\mathcal{L}\{y''\} = s^2\mathcal{L}\{y\} - sy(0) - y'(0)$
- Solving with Laplace transforms, PARTIAL FRACTIONS!
- TRANSFER FUNCTION, IMPULSE RESPONSE, CONVOLUTION.
- $\mathcal{L}\{\delta_c(t)\} = e^{-cs}$.